

Electronic Projects for Artists II

Visualizing Dynamic Systems, Chaos

Dynamic Non-Linear Systems

Most of the material for today's coding examples are drawn from explanations in the text of the book:

Complexity

by Dr. Melanie Mitchell,
a professor at Portland State
University in Oregon.



Melanie Mitchell

The scientific field of **Complex Systems** is a relatively recent one.

Complex Systems display behaviors that are not equal to the “sum of their parts” because of ***emergent phenomena:***

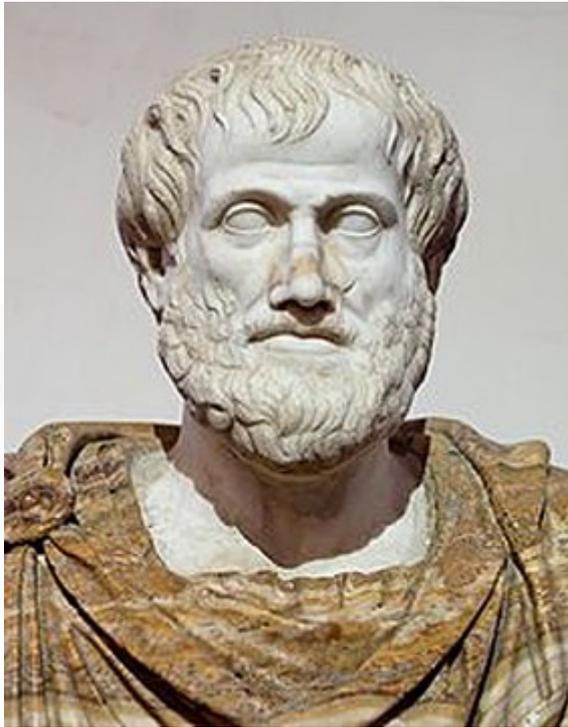
-the spontaneous development of self-organizing elements in a system that can neither be predicted nor explained by examining component parts in isolation.

Emergence and self-organization mean that no external agent is sculpting the organism: it sculpts itself.



e.g. things like flocks of birds, ant bridges and Internet trends.

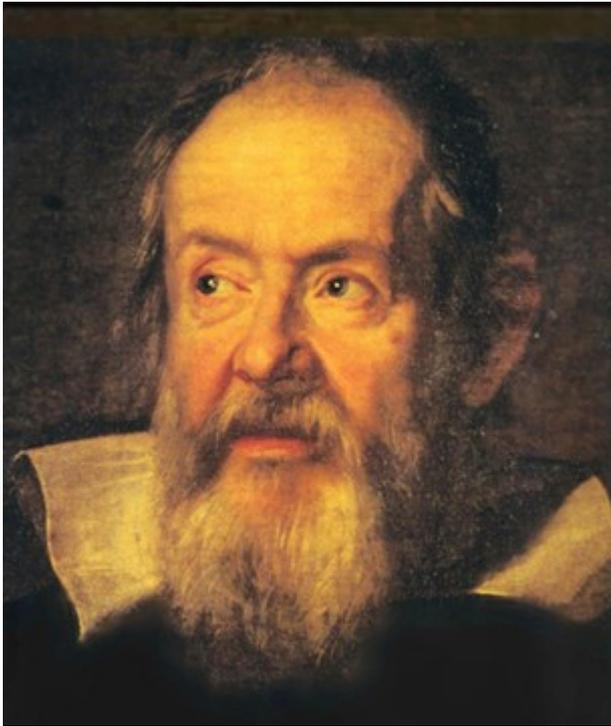
Early Roots of Dynamic Systems Theory



Aristotle (384 – 322 BCE.)

Wrote one of the first theories of motion in systems- including our solar system.

- ▶ Heavenly objects and Earthly objects move differently based on different forces
- ▶ Earthly objects move differently based on their composition (air, earth, water, fire and aether.)
- ▶ “Scientific” method based on logic and common sense, not experiment.



Galileo (1564-1642)

Pioneer of experimental, empirical science (along with Copernicus and Kepler.)

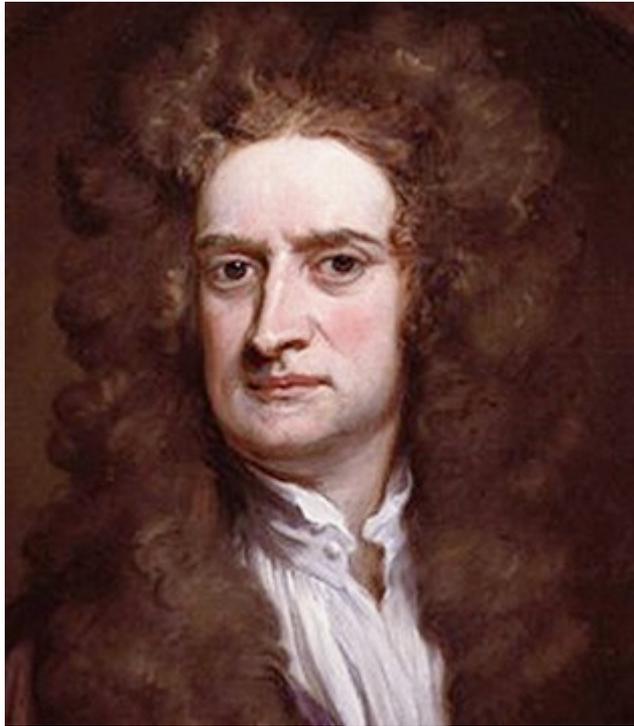
- ▶ Heavenly objects and Earthly objects move based on the same forces.
- ▶ The Earth revolves around the Sun. Stars are distant suns.
- ▶ Helped launch a revolution in Science based on empirical observation.

Reductionism

The belief that “a whole can be understood completely if you understand its parts and the nature of their sum.”

René Descartes:

(On his scientific method) “to divide all the difficulties under examination into as many parts as possible, and as many as were required to solve them in the best way. ...beginning with the simplest and most easily understood objects and gradually ascending, as it were step by step, to the knowledge of the most complex.”



Isaac Newton (1643-1727)

Perhaps most important contributor to the study of dynamic systems.

- ▶ Wrote 3 famous laws describing motion (“mechanics” in physics)
- ▶ Unified a formal description of forces effecting motion on Earth and in the heavens.
- ▶ Inspired fantasies about perfect understanding (and therefore prediction) in natural systems.



Henri Poincaré (1854-1912)

French mathematician

- ▶ Tried, failed to solve the “Three Body Problem.”

“...it may happen that small differences in the initial conditions produce very great ones in the final phenomenon. Prediction becomes impossible...”

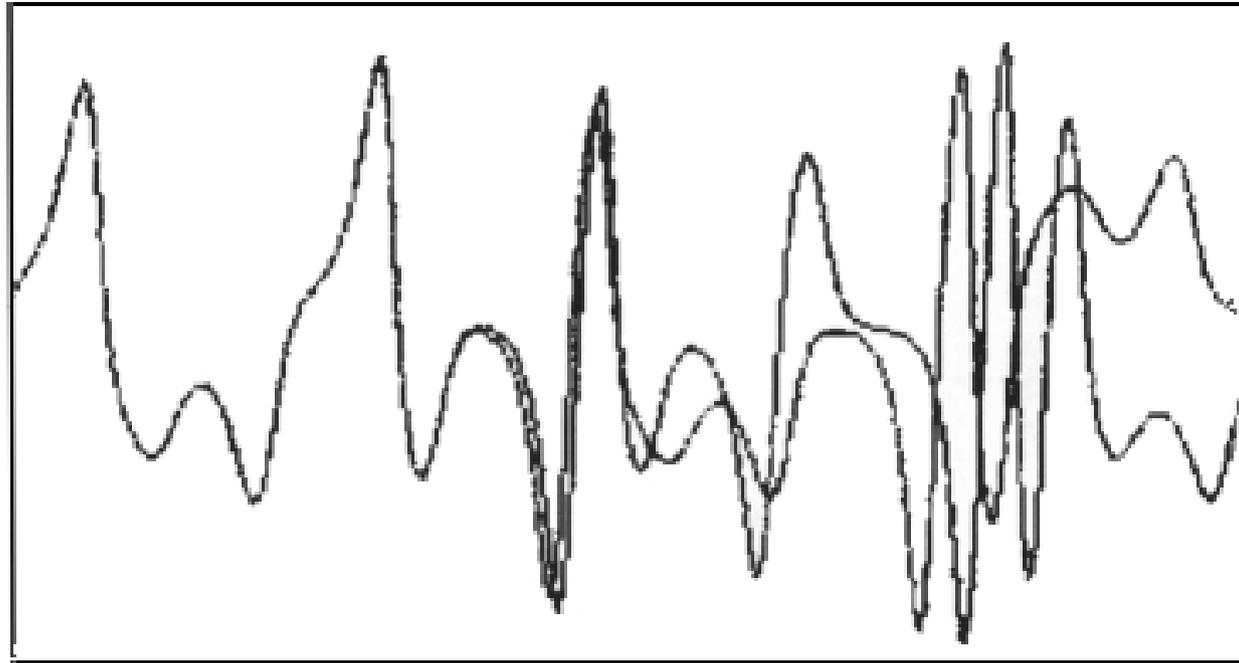
Edward Lorenz (1917-2008)

Studied weather systems at MIT

- ▶ 1963 paper: “Deterministic Nonperiodic Flow” in journal of Atmospheric Sciences is credited as the foundation of “Chaos Theory.”
- ▶ Used modern computation technology to clarify ***sensitive dependence on initial conditions.***
- ▶ Coined the term “Butterfly Effect”.

(Show: double_pendulum.mp4)

Sensitive Dependence on Initial Conditions at work:
The difference of less than 1000^{th} of one degree:



One of the properties of the new field of study called
“Chaos”, a subset “Complexity.”

Linear System Example

To get a sense of the difference between linear and non-linear systems, let's look at an example of a linear dynamic system.

A simple population growth model:

1. Every year, all the rabbits in some mythical location pair up to mate, starting in year 0 with 2 rabbits.
2. Each year after that, every bunny pair has exactly 4 offspring.
3. After raising their 4 offspring for a year, each parent couple dies.

Year 0



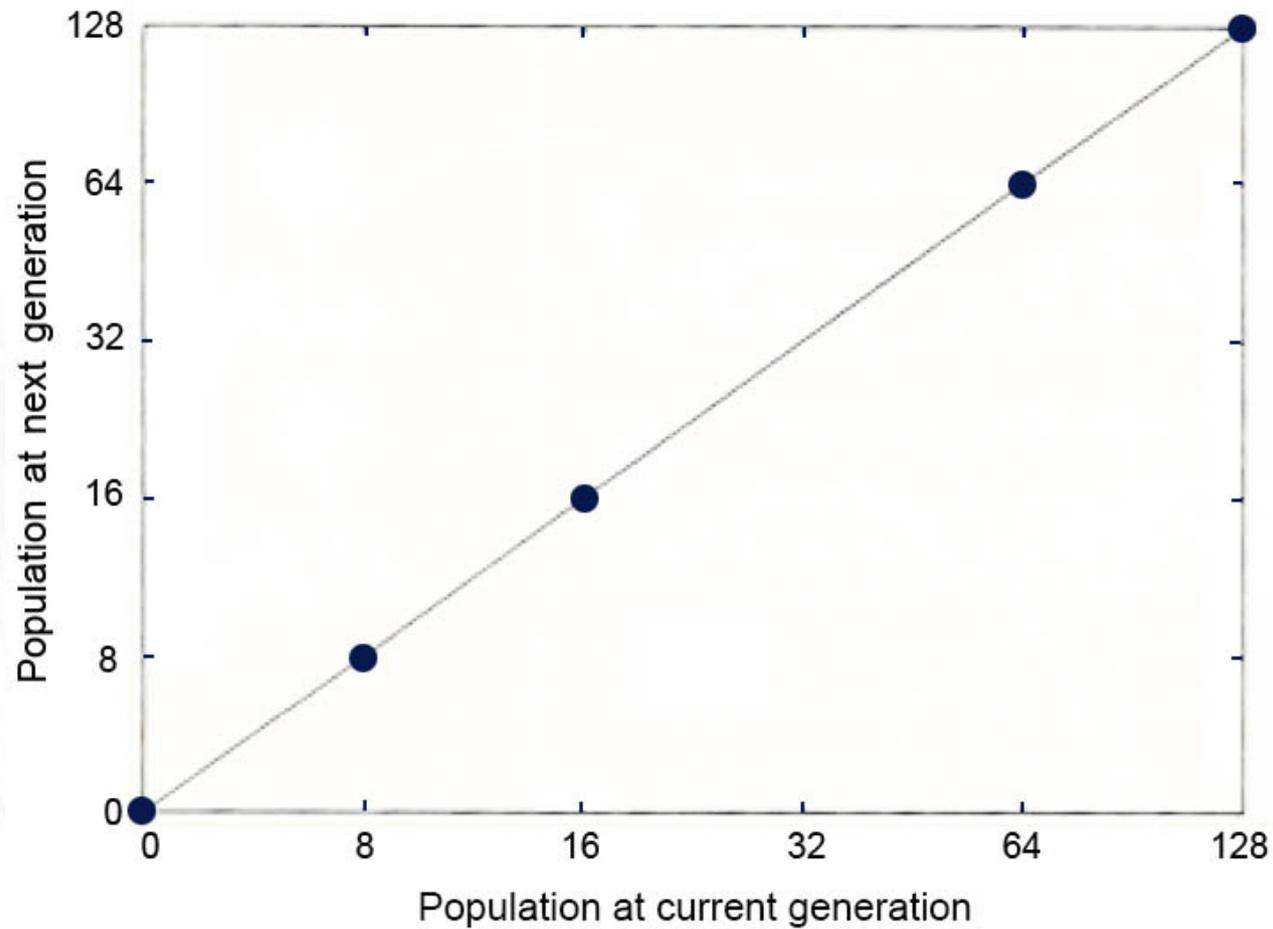
Year 1



Year 2



...eventually, they will take over planet Earth!



A plot of how the population size next year depends on the population size this year: a linear model.

You can see why the term “Linear” is used to describe this.

Growth model = an equation closer to reality.

Includes:

- > current generation size
- > the birth rate
- > the maximum upper limit of the population the habitat will support called the *carrying capacity*
- > death rate (accounting for deaths from overcrowding, predators, etc.)

Calculating the population again and again, starting each time with the previous population is called “Iterating the model.”

logistic map = simplified abstraction

- > Combines effects of birth rate and death rate into One number R .
- > Population size is replaced by a concept called the *fraction of the carrying capacity*, called x which is scaled to a range between 0 and 1.

For each time step:

$$x = Rx (1-x)$$

Multiplying the new population (x) by $(1 - x)$ scales it to prevent unlimited growth.

For each time-step we feed the new population number back into the equation:

$$x_1 = Rx_0 (1-x_0)$$

$$x_2 = Rx_1 (1-x_1)$$

$$x_3 = Rx_2 (1-x_2)$$

$$x_4 = Rx_3 (1-x_3)$$

$$x_5 = Rx_4 (1-x_4)$$

The values calculated for the previous x (population) determine the new x value (for the next population...)

Not all values for R are equally interesting.

It turns out, with values from 1 – 4 something fascinating happens...

As a system approaches chaos, it displays a characteristic behavior called “period doubling.” (see: ***Bifurcation diagram***)

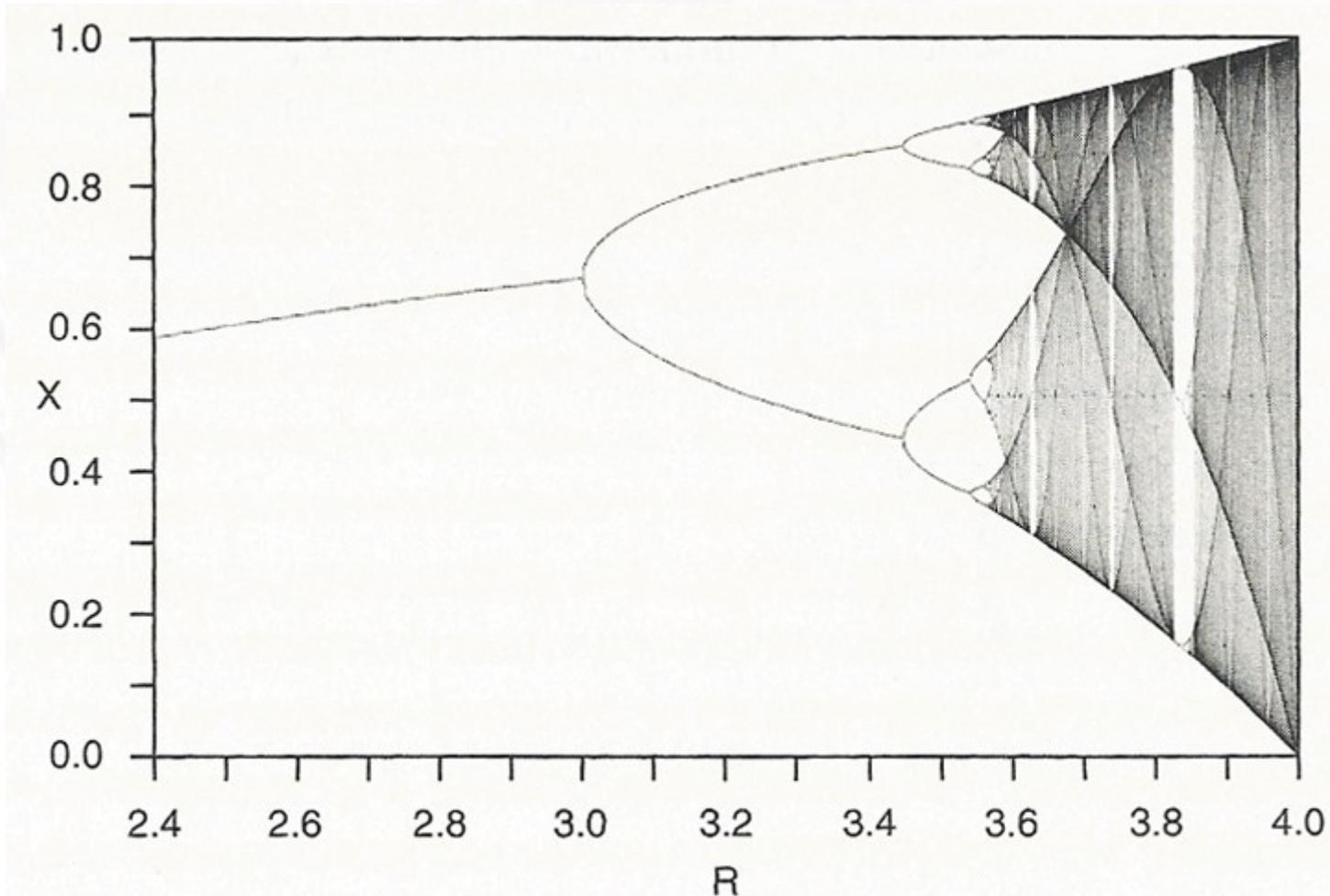
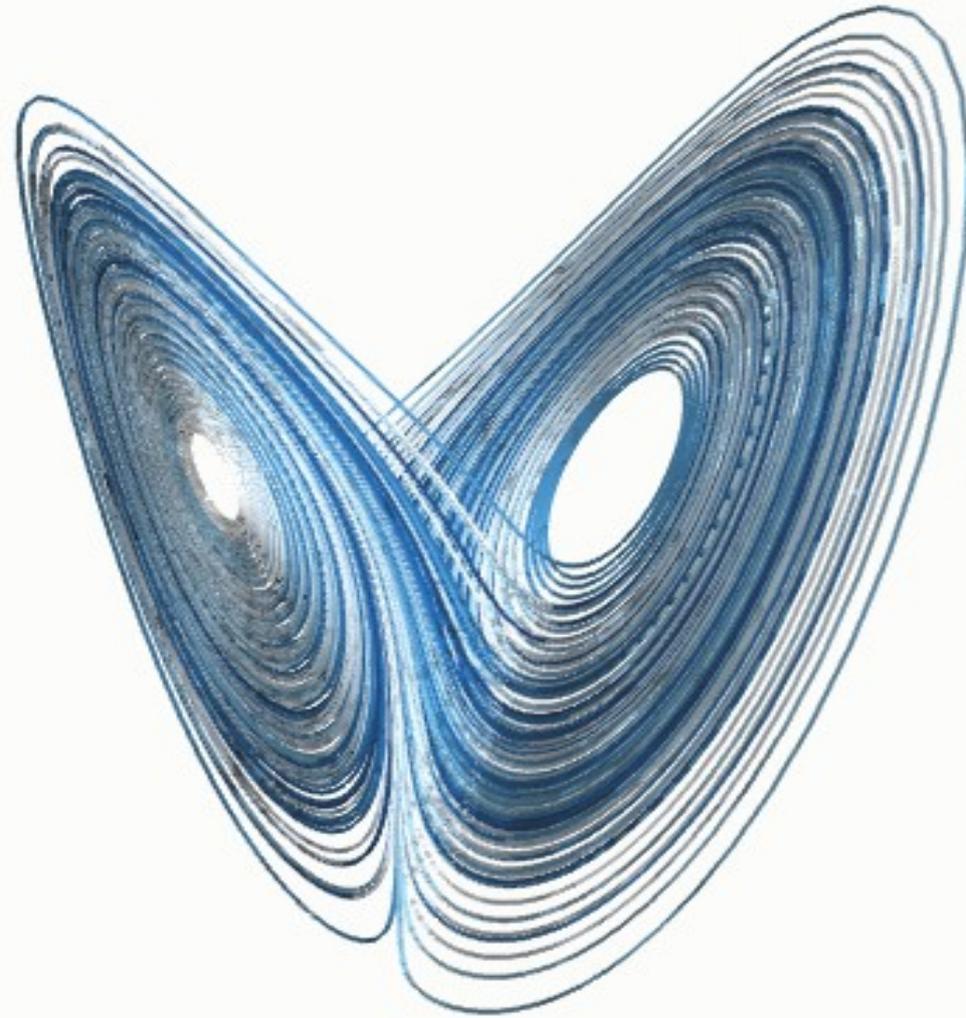


FIGURE 2.11. Bifurcation diagram for the logistic map, with attractor plotted as a function of R .



The “Lorenz Attractor” is an equation that describes possible locations in a 3D space.